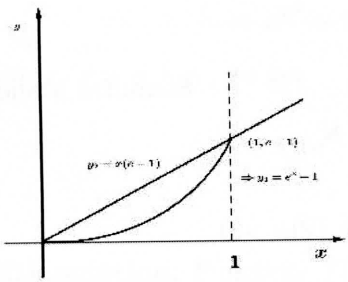

Uche Eliezer Okeke

Ningth solution. Since $e^x \geq 1 + x$ for any real x then in particular for any $x \in [0, 1]$ holds inequality

$$e^{-x} \geq 1 - x \iff 1 - e^x + xe^x \geq 0$$

(with equality iff $x = 0$).
Let $h(x) := \dfrac{e^x - 1}{x}, x \in (0, 1]$. Since $h'(x) = \dfrac{xe^x - e^x + 1}{x^2} > 0$ for any $x \in (0, 1]$ then function $h(x)$ strictly increasing on $(0, 1]$ and, therefore,

$$h(x) \leq h(1) \iff$$

$$\frac{e^x - 1}{x} \leq \frac{e^1 - 1}{1} = e - 1 \iff e^x - 1 \leq (e - 1)x$$

with equality only if $x = 1$.
Hence,

$$(e^{a^2} - 1)(e^{b^2} - 1)(e^{c^2} - 1) \leq (e - 1)^3 a^2 b^2 c^2.$$

Arkady Alt

Tenth solution. It is equivalent to show that

$$\frac{e^{a^2} - 1}{a^2} \frac{e^{b^2} - 1}{b^2} \frac{e^{c^2} - 1}{c^2} \leq (e - 1)^3$$

that is

$$\ln(e^{a^2} - 1) - 2\ln a + \ln(e^{b^2} - 1) - 2\ln b + \ln(e^{c^2} - 1) - 2\ln c \leq 3\ln(e - 1)$$